Multi-Dimensional Inverse Heat Conduction Solutions Using a "Black Box" Direct Solver

Jonathan W. Woolley, AEgis Technologies Group
Ben Blackwell, Blackwell Consulting
James V. Beck, Michigan State University

Inverse Problems Symposium
East Lansing, Michigan
June 6-8, 2010

Presentation Outline

- Background
- Inverse Heat Conduction Algorithm with “Black Box” Direct Solver
- Advantages and Disadvantages
- Direct Solver – Selection and Verification
- 2-D Verification of the IHC Code
- 3-D Verification of the IHC Code
- Concluding Remarks and Future Work
- Q & A

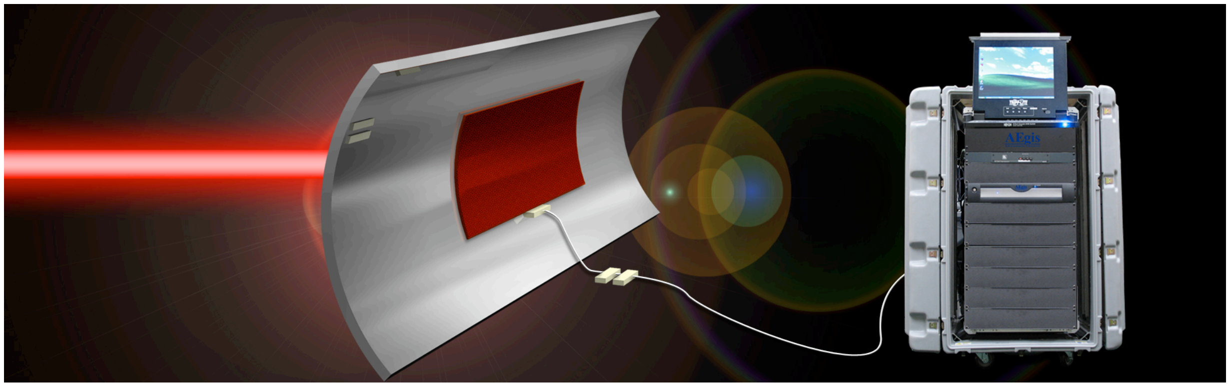
AEgis TECHNOLOGIES MICROSYSTEMS GROUP

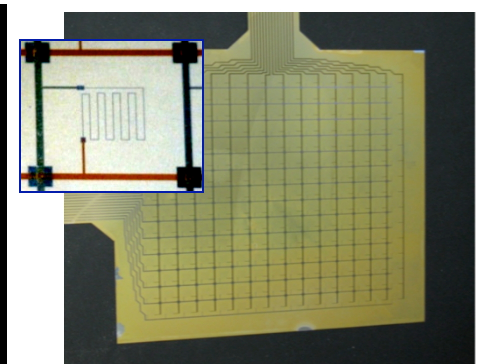
Background

- ***High Energy Laser* weapon lethality is dependent on the heat delivered to the target**

- **Directed Energy Test & Evaluation community needs an instrument that can provide accurate, high-spatial resolution measurements of heat deposited onto a target by a HEL system**
  - Off-board techniques suffer from inaccuracies
    - Target degradation products interfere with imaging systems
    - Limited engagement angle
  - Legacy on-board sensors are problematic
    - Thermocouples can not survive direct exposure
    - Large arrays and destructive testing are cost prohibitive
    - Difficult to get spatial resolution with traditional thermocouple thermometry

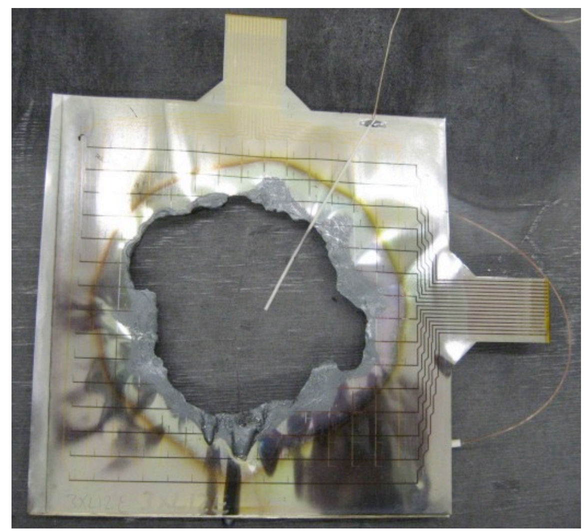
Background





Two Technology Breakthroughs:

- Disposable temperature sensor array
    - Thin film temperature sensors in a row-column array
    - Flexible, conformable substrate
    - Resistive temperature detectors (RTDs) provide fast, accurate, high resolution measurements
    - Small footprint - RTD's are about 2.5 mm^2
- An integrated IHC model that is tightly coupled with the RTD sensor array
    - Utilizes a direct heat conduction solver in conjunction with a sum of squares minimization algorithm



Inverse Heat Conduction Algorithm

AEgis Technologies MicroSystems Group

Step 1: Assume $q_i = q_{M\text{-}1}$ and march forward in time for $i = M, M + 1, \ldots, M + r - 1$

Step 2: Repeat the first step for perturbed heat flux values of $q_i^+ = q_{M\text{-}1} + \Delta q$ and $q_i^- = q_{M\text{-}1} - \Delta q$ where Δq is the finite difference heat flux perturbation

Step 3: Compute sensitivity coefficients $\frac{\partial T}{\partial q}$ at sensor nodes $j = 1, \ldots, J$ and future times $i = M, M + 1, \ldots, M + r - 1$ using

$$\frac{\partial T}{\partial q} = \frac{T(q + \Delta q) - T(q - \Delta q)}{2\Delta q}$$

Step 4: Apply the IHCP future time algorithm given by

$$\left(\sum_{i=1}^{r} \mathbf{X}^T(i)\mathbf{X}(i)\right)\Delta\mathbf{q} = \sum_{i=1}^{r} \mathbf{X}^T(i)\Delta\mathbf{Y}(M + i - 1)$$

Step 5: Using q_M from Step 4, time march the equations a single time step from $M - 1$ to M

Step 6: Advance M by 1 and *repeat steps 1 - 5*

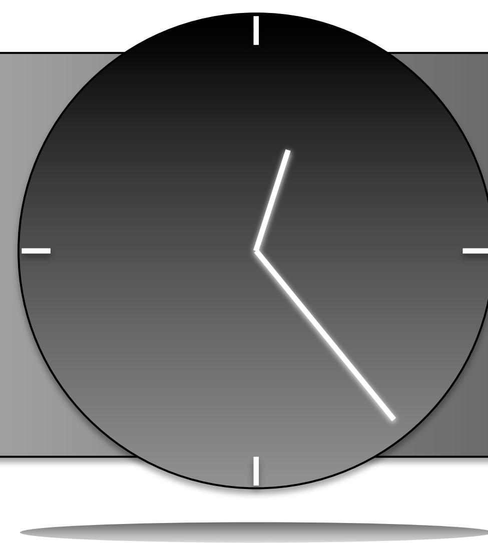
Advantages and Disadvantages

Advantages

- Multi-dimensional capabilities
- Can be implemented with Commercial Off The Shelf (COTS) Direct Solver software
- Can handle inverse problems for which special purpose IHC codes do not exist

Disadvantages

- Computation time
- The “black box” approach will be considerably slower than a well designed special purpose code



Direct Solver Selection

Direct Solver Requirements

- Transient capability
- Executions from command line
- Non-uniform initial temperature profile
- Unstructured mesh
- Temperature dependent properties

Commercial Software Evaluation

- Four COTS codes were evaluated
- Each had some shortcoming(s)

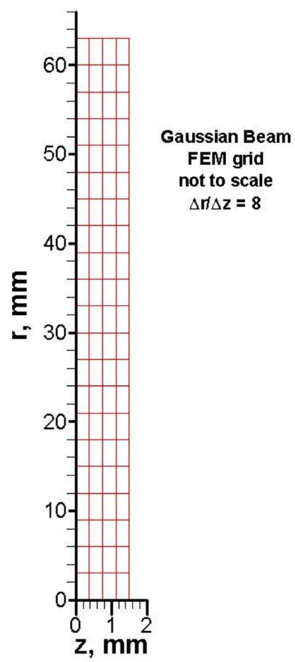
Software Selection

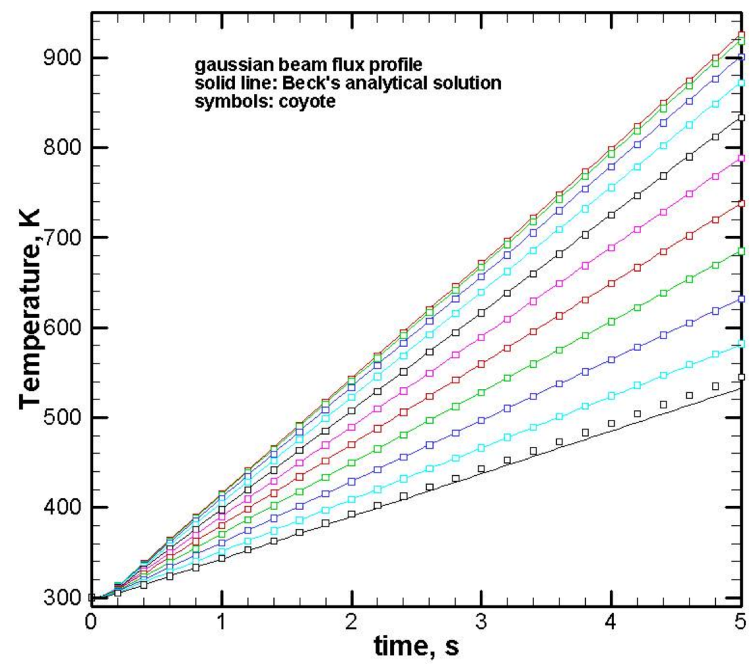
- COYOTE was selected as the direct solver
- COYOTE was developed at Sandia National Laboratories
- It is a general purpose FEM package designed for the solution of heat conduction problems

Verification of Direct Solver

Gaussian Heat Flux Profile

$$\dot{q}''(r) = \frac{P}{\pi R_{1/e}^2} \exp\left(-\frac{r^2}{R_{1/e}^2}\right)$$

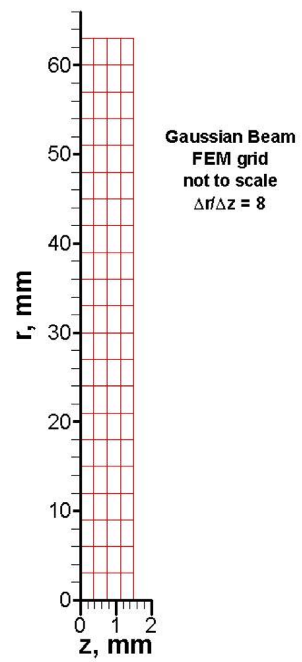


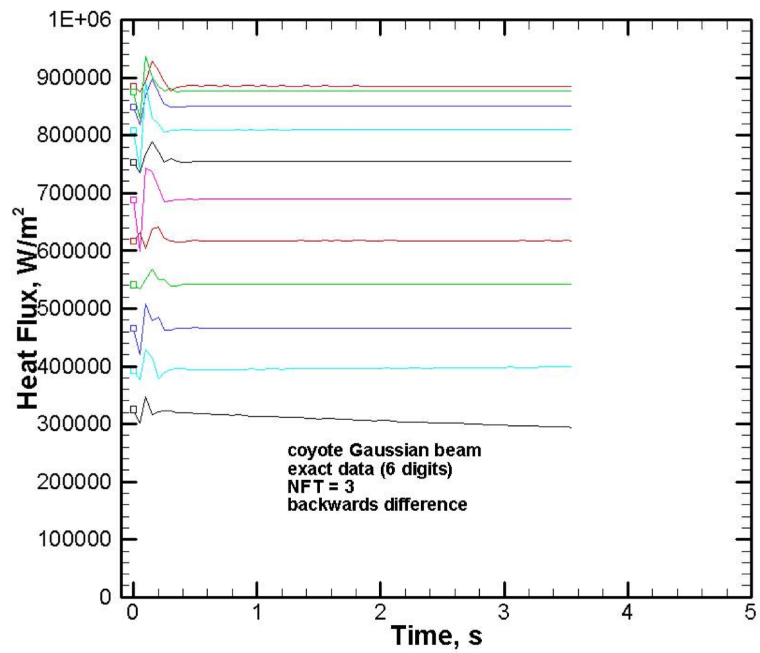


2-D IHC Verification

Gaussian Heat Flux Profile

$$\dot{q}''(r) = \frac{P}{\pi R_{1/e}^2} \exp\left(-\frac{r^2}{R_{1/e}^2}\right)$$





AEgis Technologies | MicroSystems Group

Calculation of Sensitivity Coefficients

$$\frac{\partial T}{\partial q} = \frac{T(q+\Delta q) - T(q-\Delta q)}{2\Delta q}$$

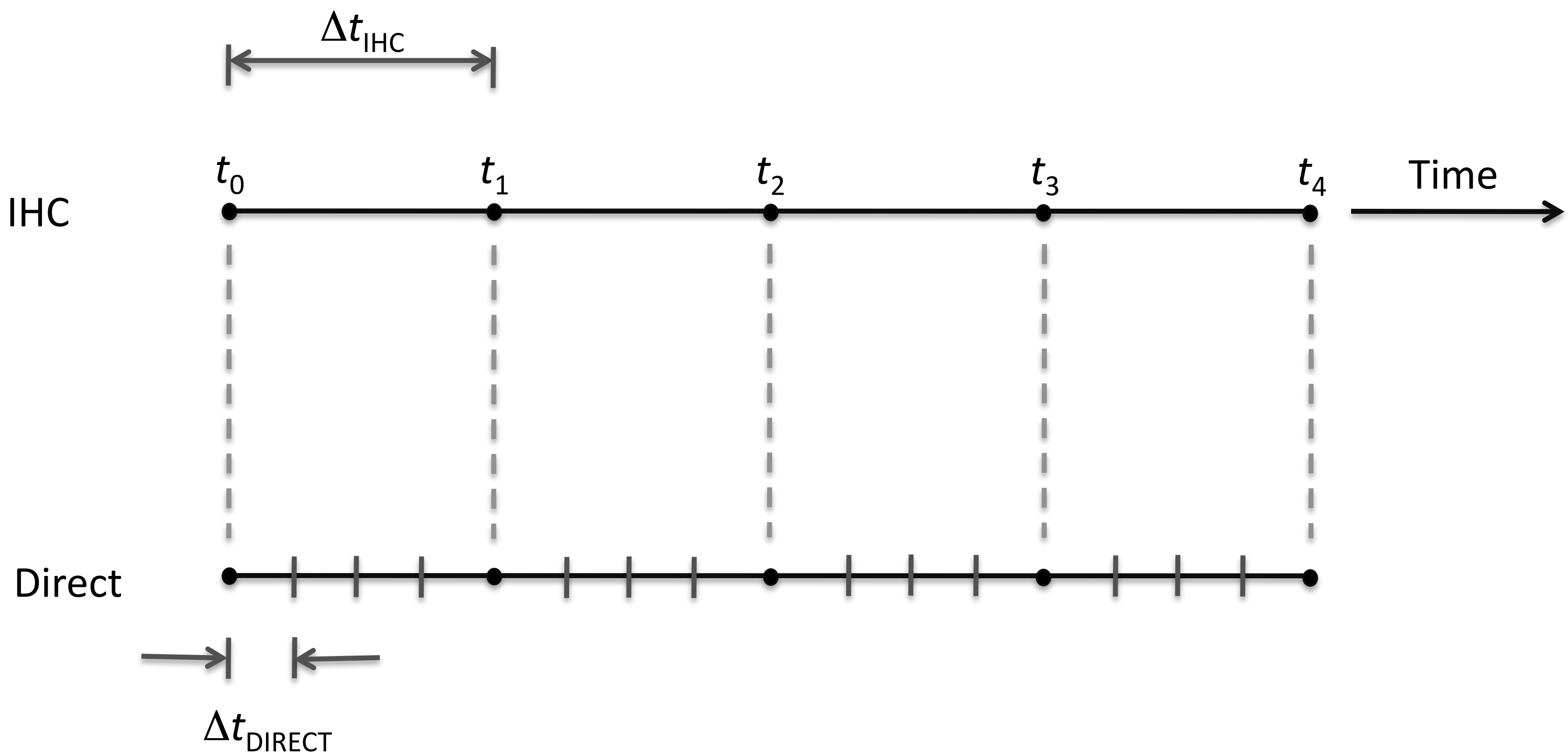
Difficulties arise when the time step is so small that a sensor does not have time to respond (particularly at early times).

At early times, the sensitivity coefficient can be very small.

For a localized-heating situation in which some of the sensors do not respond or only respond slightly, the sensitivity coefficients will be very small.

For linear problems, the finite difference step size is theoretically unimportant. For a finite precision computer, it may be important.

IHC vs. Direct Time Step



Increasing the number of direct time steps improves the sensitivity coefficients calculation

2-D IHC Verification

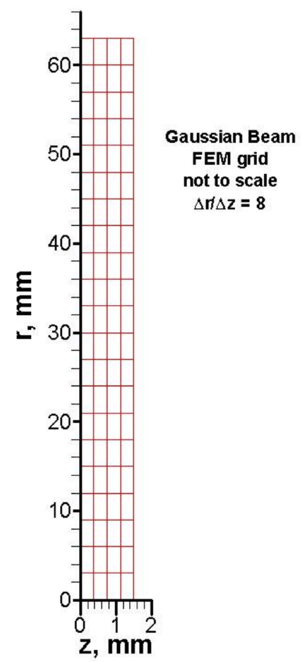
Influence of Direct Solver Time Step

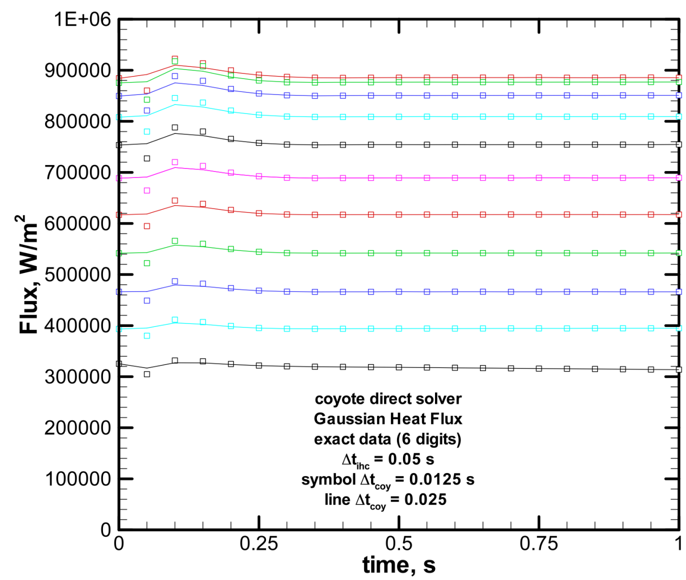
Gaussian Heat Flux Profile

Δt_{IHC} = 0.05 s

Symbol: Δt_{Coyote} = 0.0125 s

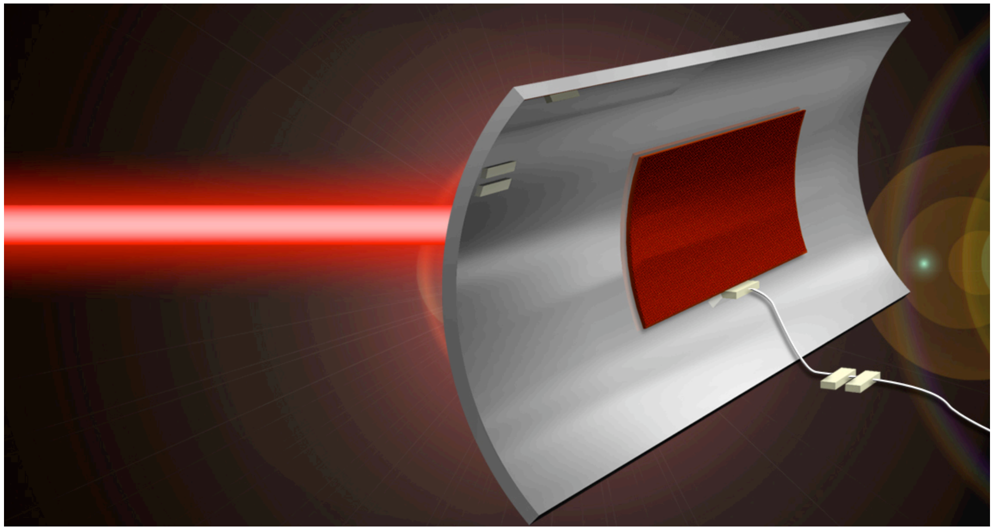
Line: Δt_{Coyote} = 0.025 s

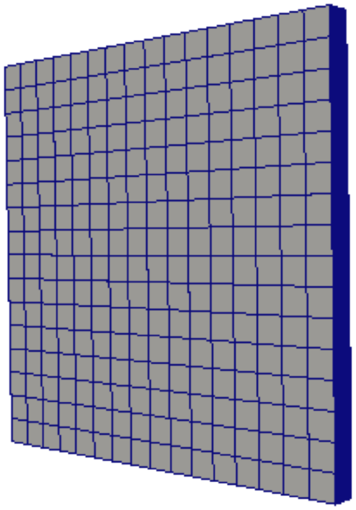




3D IHC Problem

Verification problem representative of
Gaussian laser beam heating



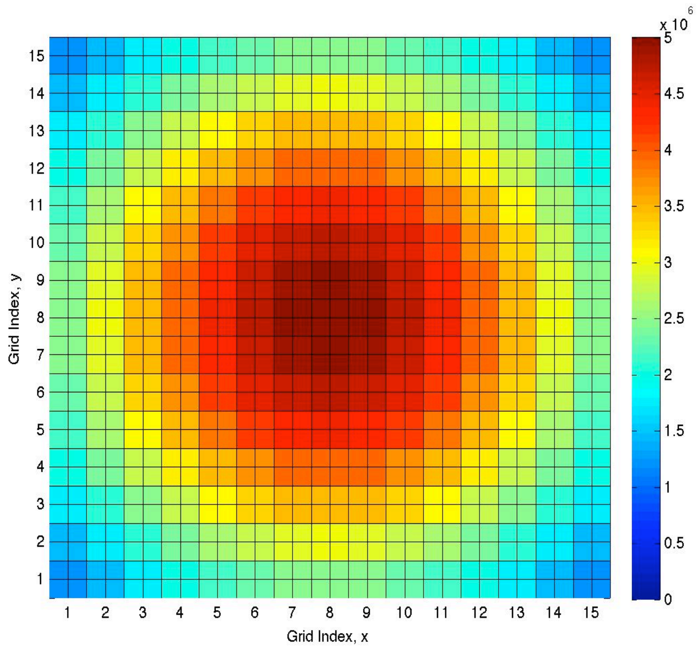


Grid used in 3-D
IHC Solutions

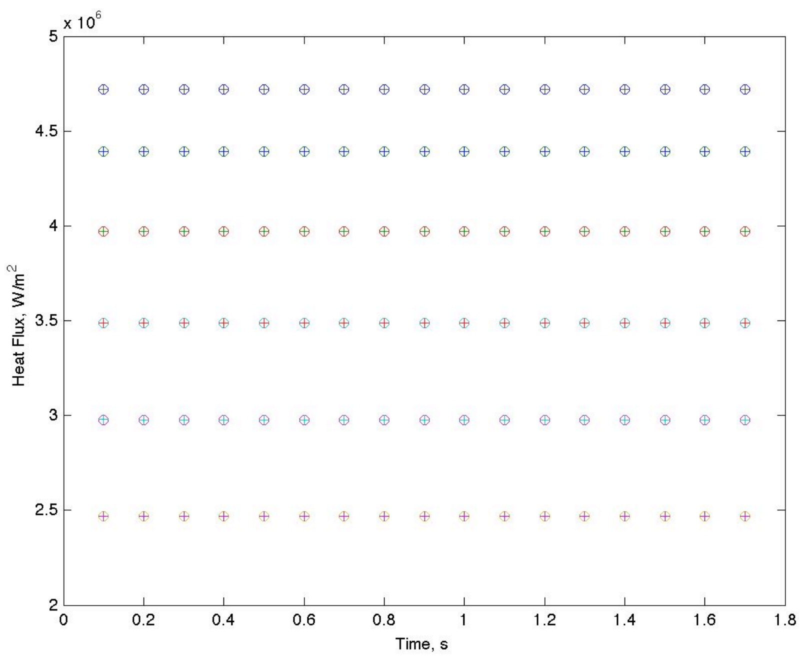
3-D IHC Verification

Influence of Direct Solver Time Step

Gaussian Heat Flux Profile
Constant with Time, Heated for 2 seconds
Errorless Data



Heat Flux Profile



IHC Heat Flux Estimates

AEgis MicroSystems Group Technologies

Future Work

Evaluation and Incorporation of Alternative Solution Techniques

- Spatial Regularization
- Singular Value Decomposition

Further Investigation of the Influence of the Direct Solver Time Step

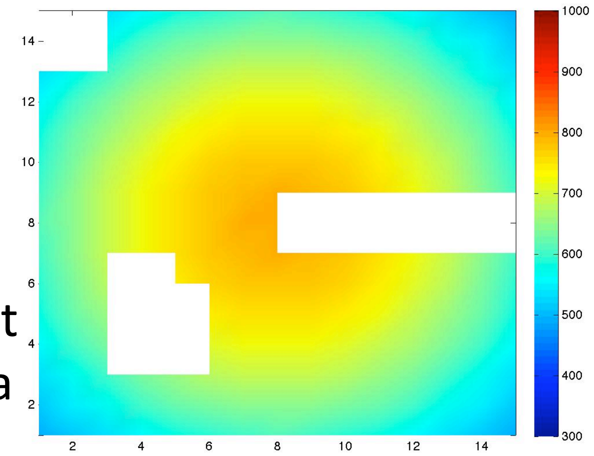
Parallelization of the Inverse Solution

Investigation of Heat Diffusion and Penetration Time Topics

- Sensor Spacing
- Under what conditions would a 1-D solution be sufficient? When is a 3-D solution necessary?

Future Work

Implement with Actual Experimental Data from *High Energy Laser* Tests



Challenges

- Sensors may fail during manufacture or transport
  - Developing more robust sensors
- Sensors may fail due to destruction of target
- Functioning sensors may yield spurious data
- Sensor survivability
  - Increase sensor temperature limits
- Calibration
  - It is desirable to calibrate through higher temperature ranges
  - Must preserve the sensors

Concluding Remarks

A versatile multi-dimensional IHC code using a "Black Box" direct solver has been successfully implemented.

- It is more computationally expensive than a special purpose IHC code
- Computation time is the trade-off for the ability to apply this approach to problems for which a special purpose code does not exist

It has been verified for

- 2-D Axisymmetric Gaussian Heat Flux Problem
- 3-D Gaussian Heat Flux Problem

NEXT STEP – Apply this approach to real world High Energy Laser experimental data. Tests have been conducted and data recorded.

AEgis Technologies MicroSystems Group

Acknowledgment and Disclaimer

This material is based upon work supported by the Test Resource Management Center (TRMC) Test and Evaluation/ Science & Technology (T&E/S&T) Program.

This project is funded by the T&E/S&T Program through the U.S. Army Program Executive Office for Simulation, Training and Instrumentation (PEO STRI) under contract numbers N61339-07-C-0010 and W900KK-08-C-0008.

Any opinions, findings and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the TRMC T&E/S&T Program and/or PEO STRI.

Questions